## Art. III.—*On the Best Form for a Balance-Beam.*

By W. C. Kernot, M.A.

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The desirable properties of a balance for accurate weighing will be found set forth in most physical text-books; and for the purposes of this paper reference may be made to Thomson and Tait's *Natural Philosophy*, Articles 430, 431, and 572; and Deschanel's *Natural Philosophy* (Everett's translation), chapter vii. From these sources the following quotations are taken :—

"The balance-beam should be as stiff as possible, and yet not very heavy."—*Thomson and Tait*, Article 430.

"Thus the stability is greater for a given load—(1) the less the length of the beam; (2) *the less its mass;* (3) the less its radius of gyration; (4) the further the fulcrum from the beam, and from its centre of gravity. With the exception of the second, these adjustments are the very opposite of those required for sensibility. Hence all we can do is to effect a judicious compromise; but the less the *mass* of the beam, the better will the balance be in *both* respects." —*Thomson and Tait*, Article 572.

"The problem of the balance, then, consists in constructing a beam of the greatest possible length and lightness, which should be capable of supporting the action of given forces without bending."—*Deschanel*, page 82.

The question, then, is to devise a form of beam which, with sufficient strength and rigidity, shall combine a mininum mass—a problem similar to that with which the engineer has to deal, on a larger scale, in designing bridges, roofs, and other framed structures—the principal difference being that while the majority of our roof and bridge frames are supported at the ends, and loaded at intermediate points, the balance-beam is supported at the centre, and loaded at the ends.

A fundamental fact that lies at the basis of all economical design is that the longitudinal strength of comparatively long and narrow pieces of ordinary materials is very large indeed, compared to the transverse strength. A wooden lath or rod that would endure a longitudinal compression of hundredweights will break with a transverse force of a few pounds; and a metal wire—a telegraph wire, for instance—that will safely bear a pull of, say, half a ton, may, when supported on points a yard apart, be bent by a force that can be easily exerted by the hand. The first point to be regarded, then, is so to arrange matters that the parts of the structure, be it bridge or balance-beam, shall be strained longitudinally, and not transversely. To fulfil this condition it is necessary that, if all in the same plane, they shall form a series of triangles, the triangle being the only polygon the form of which is absolutely fixed when the lengths of the sides are given. The simplest and lightest arrangement possible is that of two triangles, as shown in Fig. 1, where A is the fulcrum, and B and C the points from which the loads are suspended. Under the action of the loads the parts B D, D C, and A E endure tensions, and B E, E C, D A compressions, the magnitudes of which are calculable by the methods of statics on the assumption that the points B D C E are hinges. Should these points not be hinged, the actual stresses will be complicated by certain elastic actions, but to an extent that is quite unimportant when the lengths of the parts are large, compared with their transverse dimensions in plane of the beam, as is the usual case in framed structures.

Beams of a design somewhat similar to Fig. 1 in form are frequently met with. They are, however, usually open to objection on the following grounds:—

1. The bars, instead of meeting strictly at points at B and C, often terminate at different levels, as in Fig. 2, thereby losing to a large extent the benefits of the triangular system, and introducing transverse bending moment, and complicated elastic actions inimical to rigidity.

2. A number of vertical connections are introduced, adding to the mass, but not enduring any definite calculable stress.

3. Instead of one vertical diagonal D E, two bars are used, F G and H I, the portions F H and G I being bent as shown. This is a departure from all sound principles of design. If the bars F G and H I be used, as is, perhaps, desirable in order to accommodate the usual arrangement of fulcrum, then G I should be made perfectly straight and F H specially strengthened to endure the bending moment due to the upward reaction of the fulcrum. This last defect is very manifest in Figs. 42 and 43, pp. 85, 86, of *Deschanel*, representing a "balance of great delicacy."

The next question is to determine the magnitude of the angles B D E, B E D, &c., for which the mass of a beam of given strength is a minimum, and this can be accomplished by the use of the methods of the differential calculus as follows :—

Let us suppose the material of the beam to possess equal strength in compression and tension, then the beam will be symmetrical about the line BC, the lower half being the exact counterpart of the upper; let the length BC $= 2l$, DE $= 2x$, then under a given load W acting at B, the tension on BD will be

$$\mathrm{W}\frac{\mathrm{BD}}{\mathrm{DE}} = \mathrm{W}\frac{\sqrt{l^2 + x^2}}{2x}$$

The amount of material in each part of the frame will be proportional to the product of the stress into the length, therefore, the amount of material in BD will be

$$c\mathrm{W}\frac{\sqrt{l^2 + x^2}}{2x}\sqrt{l^2 + x^2} = c\mathrm{W}\frac{l^2 + x^2}{2x}$$

By symmetry the stresses on the four bars BD, DC, CE, EB will be equal, and the material required for them will be

$$4\,c\mathrm{W}\frac{l^2 + x^2}{2x}$$

The compression on DA and the tension on AE will each equal W, and the amount of material in them will be $2c\,\mathrm{W}x$

The total material in the frame is

$$4\,c\mathrm{W}\frac{l^2 + x^2}{2x} + 2c\,\mathrm{W}x \qquad (1)$$

$$= 4\,c\mathrm{W}\left(\frac{l^2 + x^2}{2x} + \frac{x}{2}\right)$$

$$= 4\,c\mathrm{W}\left(\frac{l^2}{2x} + x\right)$$

And we wish to find the value of $x$, for which the quantity in the bracket is a minimum.

Let

$$y = \frac{l^2}{2x} + x$$

$$\frac{dy}{dx} = -\frac{l^2}{2x^2} + 1 \qquad (2)$$

When $y$ is a minimum or maximum $\dfrac{dy}{dx} = 0$

Therefore—

$$-\frac{l^2}{2x^2} + 1 = 0$$

$$\frac{l^2}{2x^2} = 1$$

$$2x^2 = l^2$$

$$x = \pm\, l\, \frac{1}{\sqrt{2}}$$

Consequently the economic form is a rhombus, as in Fig. 3.

Cast steel can be obtained having a resistance both to crushing and tearing of at least 100,000 lbs. per square inch. Let such steel be used, and let the length B C be 20 inches; the sectional area of B D, D C, C E, and E B, $\frac{1}{77}$ of a square inch; and of A D, and A E, $\frac{1}{70}$ square inch; then the volume of the beam will be ·85 of a cubic inch, and its weight about one-fifth of a pound avoirdupois. The weight that would have to be placed at B and C in order to cause fracture would be, in round numbers, 1400 lbs., and a load of one-fourth of this amount, or 350 lbs., should be perfectly safe if carefully imposed. Thus we should have a beam 20 inches long, and weighing less than one-fourth of a pound, supporting safely at each end 1400 times its own weight. If made of iron, the strength of which would be from one-third to half that of the steel, it would support safely about 150 lbs. at each end; and if of brass, about 80 lbs.

Of course, the parts in compression would need to be of tubular or girder section to give lateral stiffness.

Two practical objections suggest themselves at this stage—

1. That the vertical bar D E would interfere with the necessary arrangements for the fulcrum. This difficulty may be met with by the modification shown in Fig. 4, with but inconsiderable increase of weight.

2. That this form of beam would not permit the use of the ordinary "rider." This objection may be met by stretching a fine wire from A to B and from C to D, and placing the rider on this.

In conclusion, the form of beam proposed secures the minimum mass for a given safe load, and also, by virtue of its great depth, will be exceedingly rigid. Its construction should present no special difficulty, and I see no reason why it should not be generally adopted.

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